Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

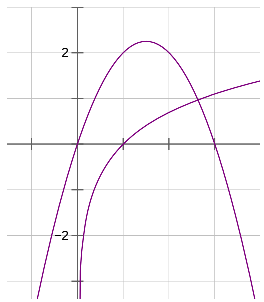
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product, power or quotient or a law of indices (on an expression that is relevant to the question)	**M1**	e.g. $\ln(e^{2x}+3)-\ln 3=\ln\left(\dfrac{e^{2x}+3}{3}\right)$ or $e^{(2x+\ln 3)}=e^{2x}e^{\ln 3}$
	State a correct equation without logs (in any form)	**A1**	e.g. $3+e^{2x}=3e^{2x}$
	Carry out correct method to solve an equation of the form $e^{2x}=a$, where $a>0$, or for solving $e^{x}=b$ $(b>0)$ if they have already taken the square root	**M1**	Allow for $x=\frac{1}{2}\ln\frac{3}{2}$. M1 can be implied by correct answer.
	Obtain answer $x=0.203$	**A1**	CAO. The question requires 3 d.p. Answer only with no working shown is 0/4.
		4	

Question	Answer	Marks	Guidance
2	Use correct double-angle formula to obtain an equation in $\cos\theta$	**M1**	e.g. $3\left(2\cos^2\theta-1\right)=3\cos\theta+2$
	Obtain $6\cos^2\theta-3\cos\theta-5=0$, or 3-term equivalent	**A1**	M1 A0 is scored if they use any correct formula for $\cos 2\theta$ and make a subsequent error.
	Solve a 3-term quadratic in $\cos\theta$ for θ	**M1**	As far as $\theta=\cos^{-1}\left(\frac{3-\sqrt{129}}{12}\right)$ if quadratic correct.
	Obtain a correct answer, e.g. $134.1°$	**A1**	Accept greater accuracy e.g. 134.1456, 225.8544.
	Obtain a second answer, e.g. $225.9°$ and no other in $[0°, 360°]$	**A1 FT**	Treat answers in radians (2.34 and 3.94) as a misread. Ignore answers outside $[0°, 360°]$. The FT is for $360°$ minus the first answer.
			Special Ruling: If they have an **incorrect** quadratic that leads legitimately to 4 solutions for θ, allow FT for $360°$ minus an answer in $(0°, 180°)$. More than 4 solutions is maximum M1 A0 M1 A0 A0. If *their* equation should have 4 solutions and the candidate only gives 3 solutions then M1 A0 M1 A0 A0. Mis-read leading to a quadratic with 4 solutions could score maximum M1 A0 M1 A1 A1 or M1 A0 M1 A1 A0 if extra/missing solution.
		5	

Question	Answer	Marks	Guidance
3	Substitute $x = \frac{1}{2}$, equate result to zero	**M1**	Or divide by $2x$–1 and equate constant remainder to zero.
	Obtain a correct simplified equation	**A1**	e.g. $\frac{1}{8}a + \frac{1}{4} + \frac{1}{2}b + 3 = 0$ or $a + 4b = -26$
	Substitute $x = -2$, equate result to 5	**M1**	Or divide by x+2 and equate constant remainder to 5.
	Obtain a correct simplified equation	**A1**	e.g. $-8a + 4 - 2b + 3 = 5$ or $8a + 2b = 2$
	Obtain $a = 2$ and $b = -7$	**A1**	WWW
		5	

Question	Answer	Marks	Guidance
4	Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	**M1**	e.g. two terms with one part of $\frac{dy}{dx} = p\cos^2 x \sin x\sqrt{\sin x} + q\frac{\cos^3 x \cos x}{\sqrt{\sin x}}$.
	Obtain correct derivative in any form e.g. $\frac{dy}{dx} = -3\cos^2 x \sin x\sqrt{\sin x} + \frac{\cos^3 x \cos x}{2\sqrt{\sin x}}$	**A1 A1**	A1 for each correct term substituted in the complete derivative.
	Equate their derivative to zero and obtain a horizontal equation with positive integer powers of sin *x* and/or cos *x* from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	**M1**	e.g. $-3\cos^2 x \sin^2 x + \frac{1}{2}\cos^4 x = 0$
	Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	**M1**	Can be awarded before the previous M1. May involve more than one trigonometric term.
	Obtain $7\cos^2 x = 6$, $7\sin^2 x = 1$, or $6\tan^2 x = 1$, or equivalent, and obtain answer *x* = 0.388	**A1**	CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. $22.2°$ is A0.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**	 $\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x - x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If ln(x) correct accept parabola for +ve y only. If ln(x) incorrect then need parabola in 3 quadrants.
	Sketch a second relevant graph, e.g. $y = 3x - x^2$, **and** justify the given statement by marking the root on the sketch or by use of a suitable comment	**B1**	
		2	
5(b)	Calculate the values of a relevant expression or pair of expressions at x = 2 and x = 2.8	**M1**	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $0.693 < 2$ and $1.03 > 0.56$ or $1.307 > 0, -0.47 < 0$ using $\sqrt{3x - \ln x}$ $0.304 > 0, -0.085 < 0$. Need to have calculated values to at least 2 sf.
		2	

Question	Answer	Marks	Guidance
5(c)	Use the iterative process correctly at least once	**M1**	
	Obtain final answer 2.63	**A1**	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	**A1**	**SC** Allow **M1 A1 A0** to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

Question	Answer	Marks	Guidance
6(a)	Correct separation of variables	**B1**	$\int e^{-y} dy = \int xe^{-x} dx$ Condone missing integral signs.
	Obtain term $-e^{-y}$	**B1**	
	Commence integration by parts and reach $\pm xe^{-x} \pm \int e^{-x} dx$	***M1**	M0 if clearly using differentiation of a product.
	Complete integration and obtain $-xe^{-x} - e^{-x}$	**A1**	
	Use $x = 0$ and $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms ae^{-y}, bxe^{-x} and ce^{-x}, where $abc \neq 0$	**DM1**	Must see working for this. In a correct solution they should have $-e^{-y} + C = -xe^{-x} - e^{-x}$ or equivalent. If they take logarithms before finding the constant, the constant must be of the right form.
	Correct solution in any form Must follow from correct working	**A1**	e.g. $-e^{-y} = -xe^{-x} - e^{-x}$ A0 if constant of integration ignored or assumed to be zero.
	Obtain final answer $y = -\ln\left((x+1)e^{-x}\right)$ from correct working	**A1**	OE e.g. $y = x - \ln(x+1)$, $y = \ln\left(\frac{e^x}{x+1}\right)$. A0 if constant of integration ignored or assumed to be zero.
		7	
6(b)	Obtain answer $(y =) 1 - \ln 2$	**B1**	Must follow from at least 6 or7 obtained in part **6(a).**
		1	

Question	Answer	Marks	Guidance
7(a)	State or imply $6xy + 3x^2 \dfrac{dy}{dx}$ as derivative of $3x^2 y$	**B1**	
	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**	Allow B1 B1 for $\left(3x^2dx+\right)6xydx+3x^2dy-3y^2dy\left[=0\right]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\dfrac{dy}{dx}$ as subject	**M1**	Allow if zero implied by subsequent working. Allow if recover from an extra $\dfrac{dy}{dx} = \ldots$ at the beginning of the left-hand side.
	Obtain $\dfrac{dy}{dx} = \dfrac{x^2+2xy}{y^2-x^2}$ correctly	**A1**	AG Accept y' for $\frac{dy}{dx}$.
		4	
7(b)	Equate numerator to zero	***M1**	Must be using the given derivative.
	Obtain $x = -2y$, or equivalent	**A1**	An equation with x or y as the subject SOI.
	Use $x^3+3x^2y-y^3=3$ to obtain an equation in x or y	**DM1**	$-8y^3+12y^3-y^3=3$ or $x^3-\frac{3}{2}x^3+\frac{1}{8}x^3=3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point (– 2, 1) and no others from solving their cubic equation	**A1**	Allow if each component stated separately. ISW.
	State the point (0, $-\sqrt[3]{3}$), or equivalent from correct work	**B1**	Accept (0, $\sqrt[3]{-3}$), or (0, – 1.44) (-1.44225). Allow if each component stated separately. ISW.
		5	

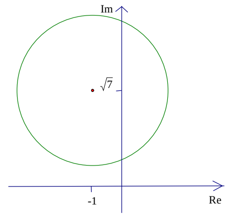
Question	Answer	Marks	Guidance
8(a)	State or imply the form $\dfrac{A}{3x-1}+\dfrac{Bx+C}{x^2+3}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 1$, $B = 0$ and $C = 3$ from correct working	**A1**	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	**A1**	
	Obtain the third value from correct working	**A1**	
		5	
8(b)	Integrate and obtain term $\dfrac{1}{3}\ln(3x-1)$	**B1 FT**	OE e.g. $\dfrac{1}{3}\ln(x-\dfrac{1}{3})$. The FT is on the value of A.
	Obtain term of the form $k\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$	**M1**	
	Obtain term $\sqrt{3}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$	**A1 FT**	OE. The FT is on the value of C.
	Substitute correct limits in an integral of the form $a\ln(3x-1)+k\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$, where $ak\neq 0$, and evaluate trigonometry	**M1**	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8-\frac{1}{3}\ln 2+\sqrt{3}\times\frac{\pi}{3}-\sqrt{3}\times\frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\dfrac{2}{3}\ln 2+\dfrac{\sqrt{3}\pi}{6}$ from correct working in part **8(b)**	**A1**	Or exact 2-term equivalent e.g. $\dfrac{1}{3}\ln 4+\dfrac{\pi}{2\sqrt{3}}$ ISW.
		5	

Question	Answer	Marks	Guidance
9(a)	Express general point of *l* or *m* in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	**B1**	
	Equate components and eliminate either λ or μ	**M1**	e.g. $\mu=\frac{2}{1-b}$, $\lambda=\frac{-1-b}{1-b}$, $\mu=\frac{-4}{2+a}$, $\lambda=\frac{a+6}{a+2}$
	Eliminate the other parameter or obtain a second expression in the first	**M1**	λ and μ are not required to be the subject of the equations.
	Show intermediate steps to obtain $2b-a=4$	**A1**	AG
	Alternative method for question 9(a)		
	Express general point of *l* or *m* in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	**B1**	
	Express *a* or *b* in terms of λ and μ	**M1**	$a=\frac{2\lambda-6}{\mu}$, $b=\frac{-1-\lambda}{\mu}$
	Use $\lambda=1-\mu$	**M1**	
	Obtain $2b-a=4$	**A1**	AG
		4	
9(b)	Using the correct process equate the scalar product of the direction vectors to zero	***M1**	$(2\mathbf{i}-\mathbf{j}-\mathbf{k}).(a\mathbf{i}+b\mathbf{j}+\mathbf{k})=0$ SOI.
	Obtain $2a-b-1=0$	**A1**	OE e.g. $2(2b-4)-b-1=0$
	Solve simultaneous equations for *a* or for *b*	**DM1**	
	Obtain $a=2$, $b=3$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(c)	Substitute found values in component equations and solve for λ or for μ	**M1**	
	Obtain answer 3**i** + **j** + 2**k** from either $\lambda = 2$ or $\mu = -1$	**A1**	Accept as coordinates or equivalent.
		2	

Question	Answer	Marks	Guidance
10(a)	Substitute $x = -1+\sqrt{7}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	***M1**	
	Use $\mathrm{i}^2 = -1$ correctly at least once and solve for k	**DM1**	$2\left(20-4\sqrt{7}\mathrm{i}\right)+3\left(-6-2\sqrt{7}\mathrm{i}\right)+14\left(-1+\sqrt{7}\mathrm{i}\right)+k=0$
	Obtain answer $k = -8$	**A1**	
			SC B1 only for those who show no working for the cube and square and obtain answer $k = -8$.
	Alternative method for question 10(a)		
	Attempt division by $\left(x+1-\sqrt{7}\mathrm{i}\right)$ as far as $2x^2+z_1x+\ldots$	***M1**	See division on next page.
	Use $\mathrm{i}^2 = -1$ correctly at least once and obtain $2x^2+z_1x+z_2$ + remainder	**DM1**	
	Obtain answer $k = -8$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(b)	State answer $-1-\sqrt{7}\mathrm{i}$	**B1**	Can be seen simply stated on its own, or in a list of roots. Allow if stated clearly in part **10(a)**.
	Carry out a method for finding a quadratic factor with zeros $-1+\sqrt{7}\mathrm{i}$ and $-1-\sqrt{7}\mathrm{i}$	**M1**	Or state $\left(x-\left(-1+\sqrt{7}\mathrm{i}\right)\right)\left(x-\left(-1-\sqrt{7}\mathrm{i}\right)\right)(2x-p)$
	Obtain x^2+2x+8	**A1**	Or obtain $\left(-1+\sqrt{7}\mathrm{i}\right)\left(-1-\sqrt{7}\mathrm{i}\right)p=-8$ Or obtain $\left(-1+\sqrt{7}\mathrm{i}\right)+\left(-1-\sqrt{7}\mathrm{i}\right)+\frac{p}{2}=-\frac{3}{2}$
	Obtain root $x=\dfrac{1}{2}$, or equivalent, via division or inspection	**A1**	Needs to follow from the working.
		4	

Question	Answer	Marks	Guidance
10(c)	Show a circle with centre $-1+\sqrt{7}\mathrm{i}$	**B1**	
	Show circle with radius 2 and centre not at the origin There needs to be some evidence of scale e.g. radius marked or a scale on the axes	**B1**	If the scales are very different from each other then B1 for centre in the correct position and B1 for an ellipse. If there is more than one circle the max score is B1.
		2	
10(d)	Carry out a complete method for calculating the maximum value of arg *z* for correct circle	**M1**	e.g. $\frac{\pi}{2}+\tan^{-1}\frac{1}{\sqrt{7}}+\frac{\pi}{4}$ Can be implied by $155.7°$.
	Obtain answer 2.72 radians	**A1**	CAO. The question requires radians.
		2	